

- ① Find the slope between ends of interval
- ② Take the derivative of function
- ③  $\text{AROC} = \text{IROC}$   
slope = derivative

$$\frac{1}{7} \times \frac{1}{3x^{2/3}}$$

$$\frac{7}{3} = \frac{3x^{2/3}}{3}$$

$$\left(\frac{7}{3}\right)^{3/2} = \left(x^{2/3}\right)^{3/2}$$

$$\left(\frac{7}{3}\right)^{3/2} = x$$

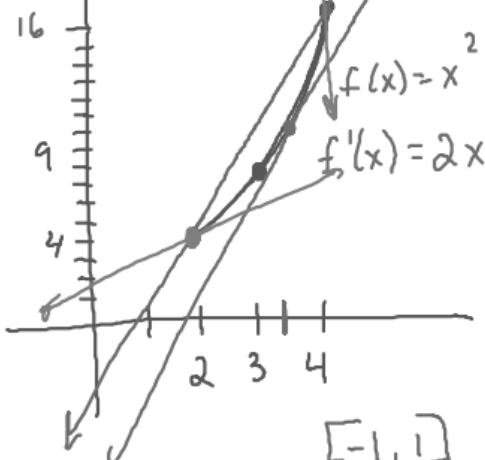
Use the Mean Value Theorem to determine where the slope of the secant line equals the slope of the tangent line

A)  $f(x) = x^2$  [2, 4]

$$(2, 4) (4, 16)$$

$$2x = 6$$

$$x = 3$$



$$[-1, 1]$$

B)  $f(x) = x^{1/3}$  [1, 8]

$$(1, 1) (8, 2)$$

$$\text{slope of secant} = \frac{2-1}{8-1} = \frac{1}{7}$$

$$f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3x^{2/3}}$$

C)  $f(x) = x^{1/3}$  [0, 1]

$$(0, 0) (1, 1)$$

$$\text{AROC} = 1$$

$$f'(x) = \frac{1}{3x^{2/3}}$$

Not differentiable  
at  $x=0$

$$\frac{1}{7} = \frac{1}{3x^{2/3}}$$

$$3x^{2/3} = 1$$

$$x^{2/3} = \frac{1}{3}$$

$$x = \left(\frac{1}{3}\right)^{3/2}$$

D)  $f(x) = x^2$  [-2, 2]

$$(-2, 4) (2, 4)$$

$$\text{slope} = 0$$

$$f'(x) = 2x$$

$$2x = 0$$

$$x = 0$$

$$f(x) = x^{1/3} [-1, 1]$$

MVT guarantees nothing  
b/c  $f(x)$  not differentiable  
at  $x=0$

Calc OK

92.

Let  $f$  be the function defined by  $f(x) = x + \ln(x)$ . What is the value of  $c$  for which the instantaneous rate of change of  $f$  at  $x = c$  is the same as the average rate of change of  $f$  over  $[2, 6]$ ?

$$f(x) = x + \ln(x) \quad [2, 6]$$

$$f'(x) = 1 + \frac{1}{x}$$

$$(2, 2 + \ln 2) \quad (6, 6 + \ln 6)$$

$$\text{slope} = \text{ARC} = \frac{6 + \ln 6 - (2 + \ln 2)}{6 - 2} = \frac{4 + \ln 6 - \ln 2}{4}$$

If  $f(x) = \cos\left(\frac{x}{2}\right)$ , then there exists a number  $c$  in the interval  $\frac{\pi}{2} < x < \frac{3\pi}{2}$  that satisfies the conclusion of the Mean Value Theorem. Find those values.

$$\left(\frac{\pi}{2}, \cos\frac{\pi}{4}\right) \quad \left(\frac{3\pi}{2}, \cos\frac{3\pi}{4}\right)$$

$$\left(\frac{\pi}{2}, \frac{\sqrt{2}}{2}\right) \quad \left(\frac{3\pi}{2}, -\frac{\sqrt{2}}{2}\right)$$

$$f'(x) = -\frac{1}{2} \sin\left(\frac{x}{2}\right)$$

$$\text{ARC} = \frac{\frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{2}}{2}\right)}{\frac{\pi}{2} - \frac{3\pi}{2}} = \frac{\left(\frac{2\sqrt{2}}{2}\right)}{\left(\frac{-2\pi}{2}\right)} = -\frac{\sqrt{2}}{\pi}$$

$$-\frac{1}{2} \sin\left(\frac{\pi}{2}\right) = -\frac{\sqrt{2}}{\pi}$$

**CALCULUS: Graphical, Numerical, Algebraic by Finney, Demana, Waits and Kennedy**  
**Chapter 8: Applications of Derivatives      8.2: L'Hopital's Rule pg. 444-452**

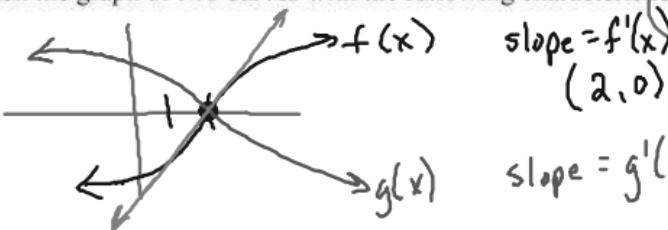
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What you'll Learn About:

How to use derivatives to find limits in an indeterminate form

Why L'Hopital's Works

Sketch the graph of two curves with the following characteristic  $f(2) = g(2) = 0$ .



a) Write the tangent line for  $f(x)$

$$y = 0 + f'(x)(x-2)$$

$$\text{c)} \lim_{x \rightarrow 2} \frac{f(x)}{g(x)} = \frac{\cancel{f(2)}}{\cancel{g(2)}} = \frac{0}{0}$$

b) Write the tangent line for  $g(x)$

$$y = 0 + g'(x)(x-2)$$

$$\lim_{x \rightarrow 2} \frac{f'(x)(x-2)}{g'(x)(x-2)} = \boxed{\frac{f'(x)}{g'(x)}} \Big|_{x=2} = \frac{f'(2)}{g'(2)}$$

$$\text{d)} \lim_{x \rightarrow 0} \frac{2x^2}{x^2} = \frac{0}{0}$$

$$2) \lim_{x \rightarrow 0} \frac{\sin(5x)}{x} = \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{4x}{2x} = \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{5\cos(5x)}{1} = \frac{5\cos(0)}{1} = 5$$

$$\lim_{x \rightarrow 0} \frac{4}{2} = 2$$

$x^{1/3}$

$$4) \lim_{x \rightarrow 1} \frac{\sqrt[3]{x-1}}{x-1} = \frac{0}{0}$$

$$\lim_{x \rightarrow 1} \frac{\frac{1}{3}x^{-2/3}}{1} = \frac{1}{3}$$

$$A) \lim_{x \rightarrow \infty} \frac{x^3-1}{4x^3-x-3}$$

$$49) \lim_{x \rightarrow 1} \frac{x^3-1}{4x^3-x-3} = \frac{0}{0}$$

$$\lim_{x \rightarrow 1} \frac{3x^2}{12x^2-1} = \frac{3}{11}$$

$$27) \lim_{x \rightarrow \infty} \frac{\ln(x^5)}{x}$$

$$35) \lim_{x \rightarrow \infty} \frac{\log_2(x)}{\log_3(x+3)}$$

$$33) \lim_{x \rightarrow 0} \frac{\sin(x^2)}{x}$$

